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3D Localisation of Target using Elevation Angle Algorithm with the use of Ground Radars

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ABSTARCT

A new novel method based on elevation angle algorithm (EAA) is proposed in this paper, to obtain 3D position of target using range and azimuth measurements of two ground 2D radars. The EAA estimates optimal target elevation angle wrt contributing radar by solving a non-linear optimisation problem using Levenberg-Marquardt method in geo-centric frame such as earth-centred-earth-fixed. The target position in geodetic frame (WGS84) is then obtained using slant range, azimuth and estimated elevation angle. The proposed method is evaluated using simulated but realistic radar data and accuracy of estimated position is found to be comparable with true position (error within acceptable limit). The method is also evaluated with real data from actual ground 2D radars and estimated target position is found to be comparable with reference navigation data (GPS) on-board of target. For each radar, corresponding Extended Kalman filter (EKF) is used to handle noisy, asynchronous measurements and to provide estimated range and azimuth at common reference time for altitude estimation using proposed EAA method. In case of real data, the estimated altitude is found to be comparable GPS altitude with error less than 5 % of true altitude**.** From the study, it is found that EAA is suitable to estimate target position using measurements from only two contributing asynchronous 2D radars in real-time as compared to some other techniques such triangulation and Trilateration where at-least three radars are required to get the position of target. This method can be useful to utilise network of vintage long range 2D radars to determine target position and to fill the gap wherever/whenever target is out of detection range of 3D radars. In addition, EAA method is compared with commonly used methodology such range only localisation and results are presented.

Keywords: Angle of arrival; Time difference of arrival; Altitude determination from primary radars; Elevation angle algorithm; Optimisation; Levenberg-Marquardt

NOMENCLATURE

ϕ_t	Latitude of the target
λ_t	Longitude of the target
h_t	Mean-sea level altitude of the target
ϕ	Latitude of the ground radar
λ	Longitude of the ground radar
h	Mean-sea level altitude of the Ground radar
r	Slant range from ground radar to target
θ	Azimuth angle from ground radar to target wrt true North
ψ	Elevation seen from ground radar to target

1. INTRODUCTION

The accurate determination of target three-dimensional (3D) position is the most important thing in aviation industry and air defence application. In general, primary surveillance radar (PSR) or two-dimensional (2D) ground radars are greatly utilised as compared to 3D ground radars due to less operational cost. Since, PSR provides only slant range and azimuth information of an Aircraft therefore the air traffic control (ATC) system usually uses the Mode C for getting Altitude and other information with the help of interrogator and transponder. The Federal Aviation Administration (FAA) mandates that all the aircraft flying within the controlled airspace (above 10000 ft) should provide Mode C altitude information[1]. It is not possible to get the altitude of target if an aircraft is not equipped with the radar transponder or even equipped but not respond to query (e.g. enemy aircraft with different intent) from ATC. In such scenarios, it becomes mandatory to utilise network of 2D ground radars in optimal way to determine target altitude accurately. The optimal way is means of selection of best combination of radars from network based on criteria for e.g. relative target-radars geometry (popularly known as dilution of precision (DoP) in case of GPS based solution) to increase the observability and to minimise the estimated altitude error.

The determination of target position using measurements from network of ground radars is popularly known as target localisation. The target localisation methods, based on type and working mode of radars, are generally categorised into the following[2]:

- Active localisation using slant range only[3,4,5]
- Passive localisation using azimuth angle only[6]

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- Combined localisation using both slant range and azimuth angle[7].

Due to higher precision in range than azimuth measurement, range based localisation provides an accurate target position but requires minimum three simultaneous contributing radars. The passive localisation using azimuth needs minimum two contributing radars but less accurate due lower precision of azimuth measurements. Combined localisation needs minimum two contributing radars and its accuracy is par with range based localisation method.

Using single 3D radar, an accurate estimation of target position can be obtained due to availability of slant range, azimuth and elevation information[8]. However, as compared to 2D radar, 3D radars are very expensive and may have lower detection range. The target localisation using single 2D radar suffers observability issue due non availability of elevation angle measurement and may work under certain assumptions/ limitations[9]. The drawback of single 2D radar and 3D radar can be overcome by utilisation of network of 2D radars for the 3D localisation of target with accuracy in par with position obtained using single 3D radar. A lots of research work has gone through over the years to develop various types target localisation techniques[10-25]. Many of conventional techniques[1,10-13] such as angle of arrival (AOA), time difference of arrival (TDOA) and altitude determination from primary radars (ADPR) used to estimate the 3D position of the target but these are having some limitations. For example, AOA has the constraint of the collinearity and TDOA[15-18] technique is not suitable when the two radar stations are at same altitude and also it produces large altitude estimation errors than AOA. AOA, TDOA and ADPR techniques used for small range applications where flat earth assumption holds good (i.e. all the radars are in same horizontal plane and their North direction are parallel to each other) but application with network of wide spread radars this assumption is not valid due earth curvature[19]. The other drawback of conventional techniques for target localisation is not to consider radar types, its working modes, number of radars and their geographical distribution which are important factors that determine the accuracy of estimated target position.

The optimisation based range only localisation or RoL[2], algebraic solution based Trilateration[20,23] and Triangulation[21,24] are the most widely used localisation methods for many practical applications. In these methods, slant range and azimuth measurements are required from at least three precisely located radars to compute the 3D position of target. Like conventional techniques[10-13], these methods do not assume flat earth model but consider Earth as an elliptical model (WGS 84) in mathematical formulation to estimate the target altitude using slant range and azimuth measurements from 2D radars. However, these methods may show degraded performance if the target goes out detection range from any one of three radars or if there is not enough/healthy physical separation found among the three radars.

Therefore, based on limitation factors for above techniques, a new and novel method named elevation angle algorithm (EAA) is proposed which falls in category of combined localisation method. The proposed method does not assume flat earth model but consider Earth as an elliptical model (WGS 84) in mathematical formulation to estimate the target altitude using slant range and azimuth measurements from 2D radars. The mathematical formulation of proposed method is given in section 2 of paper. The proposed method needs slant range and azimuth measurements from only two radars as compared to three for range only localisation, Trilateration and Triangulation methods.

Using EAA method, firstly target elevation angle w.r.t. each contributing radar is obtained by solving a non-linear optimisation problem using Levenberg-Marquardt (LM) method in geo-centric frame such as earth-centred-earth-fixed (ECEF). The target position in geodetic i.e. WGS84 frame is then obtained using slant range, azimuth and computed elevation angle. The proposed algorithm in the paper utilises measurements from two asynchronous radars, extended Kalman filter (EKF) to handle asynchronous radars and filter the noisy measurements and uses EAA method to obtain the 3D target position for single target tracking only. However, the proposed methodology can be easily extended for multiple target tracking by utilising measurements of contributing radars associated with a particular track. The associated measurements for particular track can be obtained after performing measurement to track association at each radar data processing (RDP) station and track-to-track association at fusion centre.

1.1 Problem Description

The 3D plane layout of 2D radar and target is shown in the Fig. 1. ϕ_t, λ_t and h_t are latitude, longitude and altitude of the target and ϕ, λ and h are the latitude, longitude and altitude of the 2D radar, that is position of the ground radar. SR is the slant range of the target from the ground radar and GR is the ground range from target to the ground radar. θ (Clock wise from the true north) is the azimuth angle from the ground radar to the target and ψ is the unknown elevation from the ground radar to the target. In this figure X, Y and Z axes shows the local target coordinate plane (ENU). From the figure the main objective is to find the elevation angle and target position in 3D plane. For

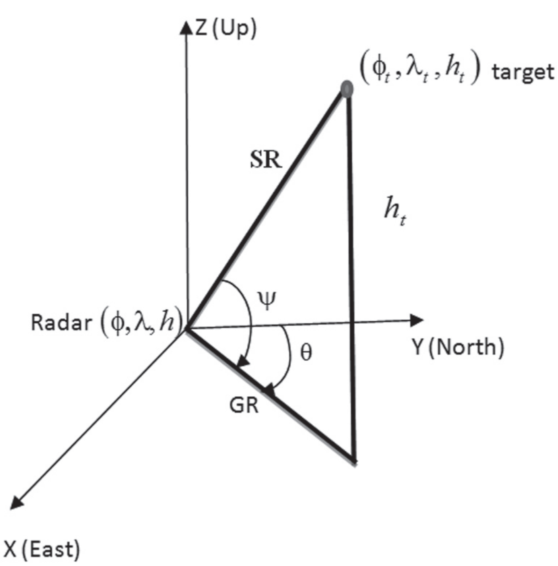


Figure 1. 3D ground plane layout of target and ground radar.

finding elevation angle seen from the radar to the target, the elevation angle algorithm (EAA) is proposed.

2. PROPOSED ELEVATION ANGLE ALGORITHM

Figure 2 shows block diagram of EAA to find the 3D position of the target. Let two ground radar stations, $[\phi_1, \lambda_1, h_1]$ and $[\phi_2, \lambda_2, h_2]$ are the positions of the radars 1 and 2 in Geodetic frame and (ϕ_t, λ_t, h_t) is target position. To find the 3D position of the target, first the elevation angles from ground radars to target has to be computed. Let ψ_1 and ψ_2 are the elevation angles from radar 1 and radar 2 to the target respectively. Elevation angles from the ground radars to target can be computed with help of the following mathematical approaches and coordinate conversions.

From Fig. 1, the target position in ENU coordination frame from ground radar as follows:

$$GR_i = r_i \cos\psi_i \tag{1}$$

$$x_{ti} = r_i \cos\psi_i \sin\theta_i \tag{2}$$

$$y_{ti} = r_i \cos\psi_i \cos\theta_i \tag{3}$$

$$z_{ti} = r_i \sin\psi_i \tag{4}$$

where, $i = 1,2$ and (x_{ti}, y_{ti}, z_{ti}) is the target position with respect to radar station 1 and 2.

2.1. Coordinate Conversion

For finding elevation angles ψ_1 and ψ_2, first convert the target position from geodetic to geo centric coordinates as follows:

Let $\left[x_{(t)}, y_{(t)}, z_{(t)}\right]$ be the target position, $\left[x_{(1)}, y_{(1)}, z_{(1)}\right]$ be the position of ground radar 1 and $\left[x_{(2)}, y_{(2)}, z_{(2)}\right]$ be the position of ground radar 2 in geo centric frame (ECEF). The coordinate conversion from geodetic to geocentric can be written in the form of state equation as follows:

$$\begin{bmatrix} x_t \\ y_t \\ z_t \end{bmatrix} = \begin{bmatrix} x_{(1)} \\ y_{(1)} \\ z_{(1)} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ b_{11} & b_{12} & b_{13} \\ c_{11} & c_{12} & c_{13} \end{bmatrix} \begin{bmatrix} r_1 \cos\psi_1 \sin\theta_1 \\ r_1 \cos\psi_1 \cos\theta_1 \\ r_1 \sin\psi_1 \end{bmatrix} \tag{5}$$

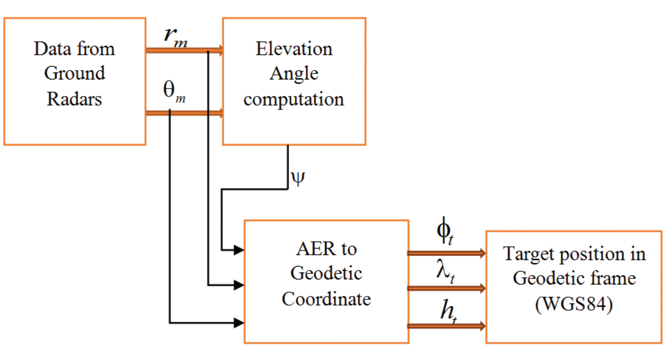


Figure 2. Work flow diagram for EAA to estimate the target 3D position.

$$\begin{bmatrix} x_t \\ y_t \\ z_t \end{bmatrix} = \begin{bmatrix} x_{(2)} \\ y_{(2)} \\ z_{(2)} \end{bmatrix} + \begin{bmatrix} a_{21} & a_{22} & a_{23} \\ b_{21} & b_{22} & b_{23} \\ c_{21} & c_{22} & c_{23} \end{bmatrix} \begin{bmatrix} r_2 \cos\psi_2 \sin\theta_2 \\ r_2 \cos\psi_2 \cos\theta_2 \\ r_2 \sin\psi_2 \end{bmatrix} \tag{6}$$

In Eqns. (5) and (6), r_1 and θ_1 are the slant range and azimuth measurements from ground radar 1. r_2 and θ_2 are the slant range and azimuth measurements from ground radar 2 to the target respectively. Generally, this slant rant range and azimuth information from radar to the target will be known from 2D radars. The unknown parameters in Eqns. (5) and (6) are the elevation angles (ψ_1 and ψ_2) from the ground radars to the target. The direction cosine matrix (DCM) elements of the equations of (5) and (6), for the ground radars 1 and 2 respectively for converting ENU Coordination frame to geo centric coordination frame are as follows:

$$\begin{aligned}
a_{11} &= \sin\lambda_1 & a_{21} &= \sin\lambda_2 \\
a_{12} &= -\sin\phi_1 \cos\lambda_1 & a_{22} &= -\sin\phi_2 \cos\lambda_2 \\
a_{13} &= -\cos\phi_1 \cos\lambda_1 & a_{23} &= -\cos\phi_2 \cos\lambda_2 \\
b_{11} &= -\cos\lambda_1 & b_{21} &= -\cos\lambda_2 \\
b_{12} &= -\sin\phi_1 \sin\lambda_1 & b_{22} &= -\sin\phi_2 \sin\lambda_2 \\
b_{13} &= -\cos\phi_1 \sin\lambda_1 & b_{23} &= -\cos\phi_2 \sin\lambda_2 \\
c_{11} &= 0 & c_{21} &= 0 \\
c_{12} &= \cos\phi_1 & c_{22} &= \cos\phi_2 \\
c_{13} &= \sin\phi_1 & c_{23} &= \sin\phi_2
\end{aligned}$$

By equating LHS of the Eqns. (5) and (6) to the same, we get

$$A\cos\psi_1 + a_{13} r_1 \sin\psi_1 - B\cos\psi_2 - a_{23} r_2 \sin\psi_2 = x_{21} \tag{7}$$

$$C\cos\psi_1 + b_{13} r_1 \sin\psi_1 - D\cos\psi_2 - b_{23} r_2 \sin\psi_2 = y_{21} \tag{8}$$

$$E\cos\psi_1 + c_{13} r_1 \sin\psi_1 - F\cos\psi_2 - a_{23} r_2 \sin\psi_2 = z_{21} \tag{9}$$

where

$$\begin{aligned}
A &= (a_{11}\sin\theta_1 + a_{12}\cos\theta_1) r_1 \\
B &= (a_{21}\sin\theta_2 + a_{22}\cos\theta_2) r_2 \\
C &= (b_{11}\sin\theta_1 + b_{12}\cos\theta_1) r_1 \\
D &= (b_{21}\sin\theta_2 + b_{22}\cos\theta_2) r_2 \\
E &= (c_{11}\sin\theta_1 + c_{12}\cos\theta_1) r_1 \\
F &= (c_{21}\sin\theta_2 + c_{22}\cos\theta_2) r_2 \\
x_{21} &= x_{(1)} - x_{(2)} \\
y_{21} &= y_{(2)} - y_{(1)} \\
z_{21} &= z_{(2)} - z_{(1)}
\end{aligned}$$

2.2. Finding Elevation Angles

In Eqns. (7), (8) and (9), the unknown variables are the elevation angles ψ_1 and ψ_2, hence by solving these equations we can get the elevation angles and then 3D position of the target can be obtained. Since these equations are non-linear trigonometric equations, therefore by using the traditional geometric methods, these equations cannot be solved. In present paper, these non-linear equations are solved by optimisation technique known as Levenberg-Marquardt (LM). The LM algorithm is considered as an interpolation between the Gauss-Newton and Gradient-Descent methods and therefore takes best of both to provide an optimal solution. Even though convergence of Trust region method is faster than LM in case of negative curvature of objective function but is less preferred due to its algorithmic complexity[25]. In MATLAB, the function 'fsolve' can be used to call in-built LM algorithm for optimisation of objective/cost function defined using Eqns. (10), (11) and (12) as given below. The objective/cost function can be written as:

$$f\left(\psi_1, \psi_2\right) = 0$$

where cost function is defined as vector in following manner:

$$A\cos\psi_1 + a_{13}r_1 \sin\psi_1 - B\cos\psi_2 - a_{23}r_2 \sin\psi_2 - x_{21} = 0 \tag{10}$$

$$C\cos\psi_1 + b_{13}r_1 \sin\psi_1 - D\cos\psi_2 - b_{23}r_2 \sin\psi_2 - y_{21} = 0 \tag{11}$$

$$E\cos\psi_1 + c_{13}r_1 \sin\psi_1 - F\cos\psi_2 - a_{23}r_2 \sin\psi_2 - z_{21} = 0 \tag{12}$$

Figure 3 illustrates a typical LM convergence of target elevation angles with respective to two contributing radars as function of iterations. It can be seen that after few iterations, the estimated elevation angle of two radars convergences to its true value. The converged value of elevation angle shall be taken to determine the target position. The convergence criteria are controlled through user defined threshold and maximum iterations.

The following MATLAB syntax is used to estimate the elevation angle of each radar:

OPTIONS = optimoptions('fsolve','Algorithm','Levenberg-Marquardt','Display','off')

$$f = @(b)\begin{bmatrix} A\cos d(b(1)) + a_{13}r_1 \sin(b(1)) - B\cos(b(2)) - a_{23}r_2 \sin(b(2)) - x_{21} \\ C\cos d(b(1)) + b_{13}r_1 \sin(b(1)) - D\cos(b(2)) - b_{23}r_2 \sin(b(2)) - y_{21} \\ E\cos d(b(1)) + c_{13}r_1 \sin(b(1)) - F\cos(b(2)) - c_{23}r_2 \sin(b(2)) - z_{21} \end{bmatrix}$$

[*b*, FVAL, EXITFLAG, OUTPUT] = *f*solve (*f*, *b*0, OPTIONS);

where *b*0 (1x2) is initial guess of elevation angle of two radars, *b* (1x2) is estimated elevation angle of the two radars, '*f*' is the cost function defined using Eqns. (10), (11), and (12).

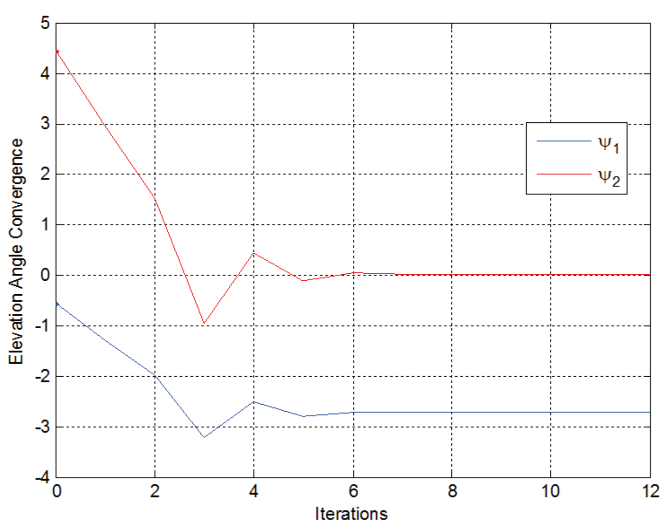


Figure 3. Illustration of target elevation angles convergence.

2.3 Coordinate conversion from AER to Geodetic frame

Slant range, azimuth and computed elevation angle of respective contributing radar are transformed to ECEF frame by using the Eqns. (5) or (6). Then the target position from ECEF to geodetic frame is computed using method provided in[26].

Case 1: Radar measurements without considering noise

In this case let the measurements (r_1, θ_1 and r_2, θ_2) of 2D radar are not corrupted by additive white Gaussian noise (AWGN) to hypothetically evaluate the performance of EAA for noise free case. Hence by solving the Eqns. (10), (11) and (12) the required elevation angles can be directly computed.

Case 2: Considering Noise Propagation to Radar Measurements

In general, the slant range and azimuth measurements come from ground radars are corrupted with AWGN. The slant range and azimuth measurements with independent AWGN are

$$r_1^m = r_1 + r_{1n} \tag{13}$$

$$\theta_1^m = \theta_1 + \theta_{1n} \tag{14}$$

$$r_2^m = r_2 + r_{2n} \tag{15}$$

$$\theta_2^m = \theta_2 + \theta_{2n} \tag{16}$$

where r_1^m and θ_1^m are the slant range and azimuth measurements of radar 1, r_2^m and θ_2^m are the measurements of radar 2, r_{1n}, r_{2n} are the noise propagated to slant range measurements of radar 1 and 2 respectively and θ_{1n}, θ_{2n} are noise propagated to azimuth measurements of radar 1 and 2 respectively. The typical value of standard deviation of realistic noise is mentioned in section 3 of the paper.

In place of r_1, θ_1 and r_2, θ_2 in Eqns. (7), (8) and (9) place the noisy measurements of ground radars r_1^m, θ_1^m and r_2^m, θ_2^m. The nonlinear trigonometric equations with the radar measurements corrupted by AWGN are as follows

$$A\cos\psi_1 + a_{13}r_1^m \sin\psi_1 - B\cos\psi_2 - a_{23}r_2^m \sin\psi_2 - x_{21} = 0 \tag{17}$$

$$C\cos\psi_1 + b_{13}r_1^m in\psi_1 - D\cos\psi_2 - b_{23}r_2^m \sin\psi_2 - y_{21} = 0 \tag{18}$$

$$E\cos\psi_1 + c_{13}r_1^m \sin\psi_1 - F\cos\psi_2 - a_{23}r_2^m \sin\psi_2 - z_{21} = 0 \tag{19}$$

where,

$$A = \left(a_{11}\sin\theta_1^m + a_{12}\cos\theta_1\right)r_1^m$$

$$B = \left(a_{21}\sin\theta_2^m + a_{22}\cos\theta_2\right)r_2^m$$

$$C = \left(b_{11}\sin\theta_1^m + b_{12}\cos\theta_1\right)r_1^m$$

$$D = \left(b_{21}\sin\theta_2^m + b_{22}\cos\theta_2\right)r_2^m$$

$$E = \left(c_{11}\sin\theta_1^m + c_{12}\cos\theta_1\right)r_1^m$$

$$F = \left(c_{21}\sin\theta_2^m + c_{22}\cos\theta_2\right)r_2^m$$

2.4 Extended Kalman Filter

The computed elevation angles, ψ_1 and ψ_2 by solving

Eqns. (17), (18) and (19), will be noisy due non-linear propagation of measurement noise. The usage of computed elevation angles from direct radar measurements will lead to inaccurate estimation of target position. In present paper, it is proposed to use Extended Kalman Filter (EKF) to handle noisy radar measurements. The filtered radar measurements using EKF are used to compute elevation angle and to find the target position. The following approaches are proposed to use EKF to handle the radar measurements:

2.4.1 Approach 1: 2D EKF

In this approach the slant range and azimuth measurements of ground radars are filtered by using their respective 2D EKF. The state model used is a constant velocity model in local East-North (EN) frame and measurements are in polar frame i.e. slant range and azimuth angle.

The constant velocity (CV) state model and radar measurement model is given by

State Model:

$$X_{k+1} = F_{CV} X_k + G_{CV} w_k$$

where, X is the state vector of target given defined as $\begin{bmatrix} x_N & v_N & y_E & v_E \end{bmatrix}$ and w is additive white Gaussian process noise. x_N and y_E is target position in North & East direction, v_N and v_E is target velocity in North and East direction. F_{CV} and G_{CV} is state transition and process noise gain matrix respectively given by

$$F_{CV} = \begin{bmatrix} 1 & dt & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & dt \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } G_{CV} = \begin{bmatrix} \frac{dt^2}{2} \\ dt \\ \frac{dt^2}{2} \\ dt \end{bmatrix}$$

where dt scan time interval.

Measurement Model:

The measurement vector Z_m is given by

$$Z_m = \left[slant\_range \quad Azimuth \right]$$

$$slant\_range = \sqrt{x_N^2 + y_E^2}$$

$$Azimuth = \tan^{-1}\left(\frac{y_E}{x_N}\right)$$

This approach is an approximate solution to target tracking due to assumption that ground range is same as slant range. But in reality this assumption may not be correct if target is flying close to radar i.e. when significant elevation angle is developed. This approach can be used to handle asynchronous radars and therefore useful for practical application. Figure 4 shows the block diagram of approach 1.

In most of the practical application of multi-target tracking in civil/defence application, the life span of target tracked are of very short duration. Therefore, it is mandate while designing EKF that its settling time shall be very fast theoretically not more than twice of dimension of filter state. To achieve fast convergence of filter state, the state error covariance is initialised using following equations[13,27,28]:

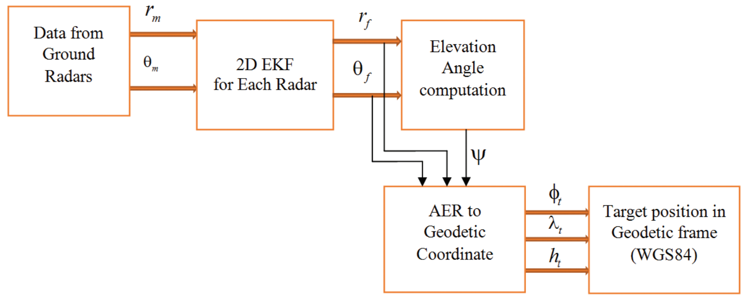


Figure 4. Block diagram of approach 1, 2D EKF.

$$P_0 = \begin{bmatrix} R_{11} & R_{12} & 0 & 0 \\ R_{21} & R_{22} & 0 & 0 \\ 0 & 0 & R_{33} & R_{34} \\ 0 & 0 & R_{43} & R_{44} \end{bmatrix}_{4X4} \tag{20}$$

The elements are as follows

$$R_{11} = \left(\lambda_\theta^{-2} - 2\right) r_m^2 \cos^2 \theta_m + \frac{1}{2}\left(r_m^2 + \sigma_r^2\right)\left(1 + \lambda_\theta^{'} \cos 2\theta_m\right)$$

$$R_{12} = R_{21} = \frac{R_{11}}{dt}$$

$$R_{22} = 2\frac{R_{11}}{dt^2}$$

$$R_{33} = \left(\lambda_\theta^{-2} - 2\right) r_m^2 \sin^2 \theta_m + \frac{1}{2}\left(r_m^2 + \sigma_r^2\right)\left(1 - \lambda_\theta^{'} \cos 2\theta_m\right)$$

$$R_{34} = R_{43} = \frac{R_{33}}{dt}$$

$$R_{44} = 2\frac{R_{33}}{dt^2}$$

where r_m = initial value of slant range measurement, θ_m = initial value of azimuth angle measurement , λ_θ and $\lambda_\theta^{'}$ are the compensation factors for exact de-biasing are as follows: $\lambda_\theta = e^{\frac{-\sigma_\theta^2}{2}}$ and $\lambda_\theta^{'} = \lambda_\theta^4$, and σ_θ^2 is the standard deviation for the azimuth noise error, σ_r^2 is the standard deviation for the slant range noise error and dt is the scan time interval of radar.

2.4.2 Approach 2: 3D EKF

Figure 5 shows the block diagram of approach 2.

The drawback of approach 1 is overcome by use of 3D EKF where measurements are slant range, azimuth and computed elevation angle from these measurements. The other advantage is that target tracking is done in ENU frame which supposed to more accurate than 2D tracking. This approach is useful for synchronous radars only i.e. when their time of detection (TOD) are same. To achieve fast convergence of filter state, the state error covariance is initialised using following equations[13,27,28]:

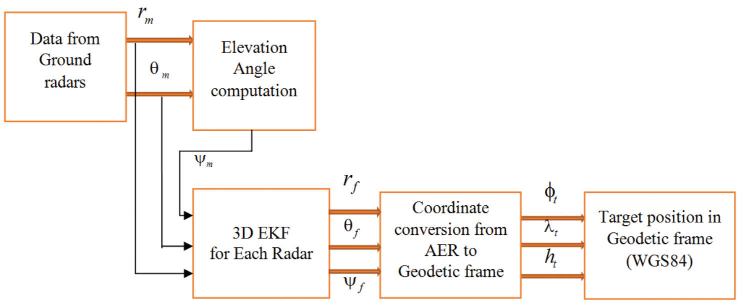


Figure 5. Block diagram of approach 2, 3D EKF.

$$P_0 = \begin{bmatrix} R_{11} & R_{12} & 0 & 0 & 0 & 0 \\ R_{21} & R_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & R_{33} & R_{34} & 0 & 0 \\ 0 & 0 & R_{43} & R_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & R_{55} & R_{56} \\ 0 & 0 & 0 & 0 & R_{65} & R_{66} \end{bmatrix}_{6X6} \quad (21)$$

The elements for this covariance matrix is as follows

$$R_{11} = \left(\left(\lambda_\theta \lambda_\psi\right)^{-2} - 2\right) r_m^2 \cos^2\theta_m \cos^2\psi_m + \frac{1}{4}\left(r_m^2 + \sigma_r^2\right)$$
$$\left(1 + \lambda_\theta' \cos 2\theta_m\right)\left(1 + \lambda_\psi' \cos 2\psi_m\right)$$

$$R_{12} = R_{21} = \frac{R_{11}}{dt}$$

$$R_{22} = 2\frac{R_{11}}{dt^2}$$

$$R_{33} = \left(\left(\lambda_\theta \lambda_\psi\right)^{-2} - 2\right) r_m^2 \sin^2\theta_m \cos^2\psi_m + \frac{1}{4}\left(r_m^2 + \sigma_r^2\right)$$
$$\left(1 - \lambda_\theta' \cos 2\theta_m\right)\left(1 + \lambda_\psi' \cos 2\psi_m\right)$$

$$R_{34} = R_{43} = \frac{R_{33}}{dt}$$

$$R_{44} = 2\frac{R_{33}}{dt^2}$$

$$R_{55} = \left(\lambda_\psi^{-2} - 2\right) r_m^2 \sin^2\psi_m + \frac{1}{2}\left(r_m^2 + \sigma_r^2\right)\left(1 - \lambda_\psi' \cos 2\psi_m\right)$$

$$R_{56} = R_{65} = \frac{R_{55}}{dt}$$

$$R_{66} = 2\frac{R_{55}}{dt^2}$$

r_m = initial value of slant range measurement

θ_m = initial value of azimuth angle measurement

ψ_m = initial value of computed elevation angle

λ_θ and λ_θ' are the compensation factors for exact debiasing for cosine angle of bearing or azimuth angle and $\lambda_\psi, \lambda_\psi'$ are the elevation angle compensation factors for exact debiasing for cosine angle of the computed elevation angle and these are as follows:

$\lambda_\theta = e^{\frac{-\sigma_\theta^2}{2}}$ and $\lambda_\theta' = \lambda_\theta^4$, and σ_θ^2 is the standard deviation for the azimuth noise error and σ_r^2 is the standard deviation for the slant range noise error. $\lambda_\psi = e^{\frac{-\sigma_\psi^2}{2}}$ and $\lambda_\psi' = \lambda_\psi^4$, σ_ψ^2 the standard deviation for the computed elevation angle noise error and dt is the scan time interval of radar.

2.5 Implementation Steps for EAA

Following are the steps for real-time implementation of EAA for single target:

- Receive the associated measurement data (slant range and azimuth) of two asynchronous 2D radars on communication network of particular track
- Remove the biases and scale factor from measurements
- Use EKF to filter the bias/scale factor corrected noisy measurements and handle the asynchronous radars to bring the measurements to user defined common reference time.
- Use bias/scale factor corrected and filtered measurements of two radars, radars location and apply optimisation technique (on cost function defined using Eqns. (10), (11), and (12)) to estimate the elevation angle of each radar
- Using coordinate conversion technique, estimate target position in WGS84 using any of contributing radar's bias/scale factor corrected filtered measurement, estimated elevation angle and its location.
- Update the reference time by user defined time interval and repeat the above steps.
- Display the target position in WGS84 (i.e. latitude, longitude and altitude above mean-sea level) to user at every reference time update at fusion centre

3. SIMULATION RESULTS

The target simulation was carried out using *Flight gear* open source software to generate target latitude, longitude and altitude. The target was flown around 30000 feet from mean-sea level with a constant ground velocity of 900 km/h. The two radars are placed far from each other around the target flown area with radar 1 at 12.0660 deg latitude, 82.1046 longitude and antenna height of 200 m from mean-sea level and radar 2 at 11.5276 deg latitude, 78.3505 deg longitude and 230 m. The scan rate of these radars is 15 rounds per minute (RPM). The slant range and azimuth measurements are generated using target position and respective radar location. Since it is a simulated case, therefore parameters such as minimum/maximum detection range, field of view are not considered and it is assumed that radars are synchronous i.e. their Time of detection (TOD) are same. The true measurements are corrupted with additive white gaussian noise (AWGN) with realistic 5 m and 0.0275 deg standard deviation in slant range and azimuth respectively for radar 1. For radar 2, standard deviation of 17 m and 0.0487 deg kept for slant range and azimuth respectively. The true target elevation angle is also simulated for each radar to compare it with computed elevation angle from EAA method.

Figure 6 shows true elevation for each radar compared with computed elevation angle from EAA using noise free radar measurements.

It can be seen from the Fig. 6 that elevation angle error is of the order of 10^{-7} deg which is very small. Figure 7(a) shows the altitude error of the target with a very small error of less than 3 mm. Figure 7(b) shows the Root sum square position error (RSSPE) in ECEF frame with the very small error showing very accurate 3D position estimation using the proposed method. It is concluded that for noise free case the EAA performance is ideal.

The performance of EAA is also evaluated when direct noisy radar measurements are used to compute elevation angle. Figures 8 and 9 show the comparison of elevation angles and altitude error respectively. It can be seen that measurement noise is non-linearly propagated in computed elevation angle and its standard deviation varies depending on target-radar geometry. The elevation angle error is significantly large which in turn reflected on degraded estimated target altitude and RSSPE (see Figs. 9(a) and 9(b)). It is therefore recommended not to use noisy measurements directly to calculate elevation angle.

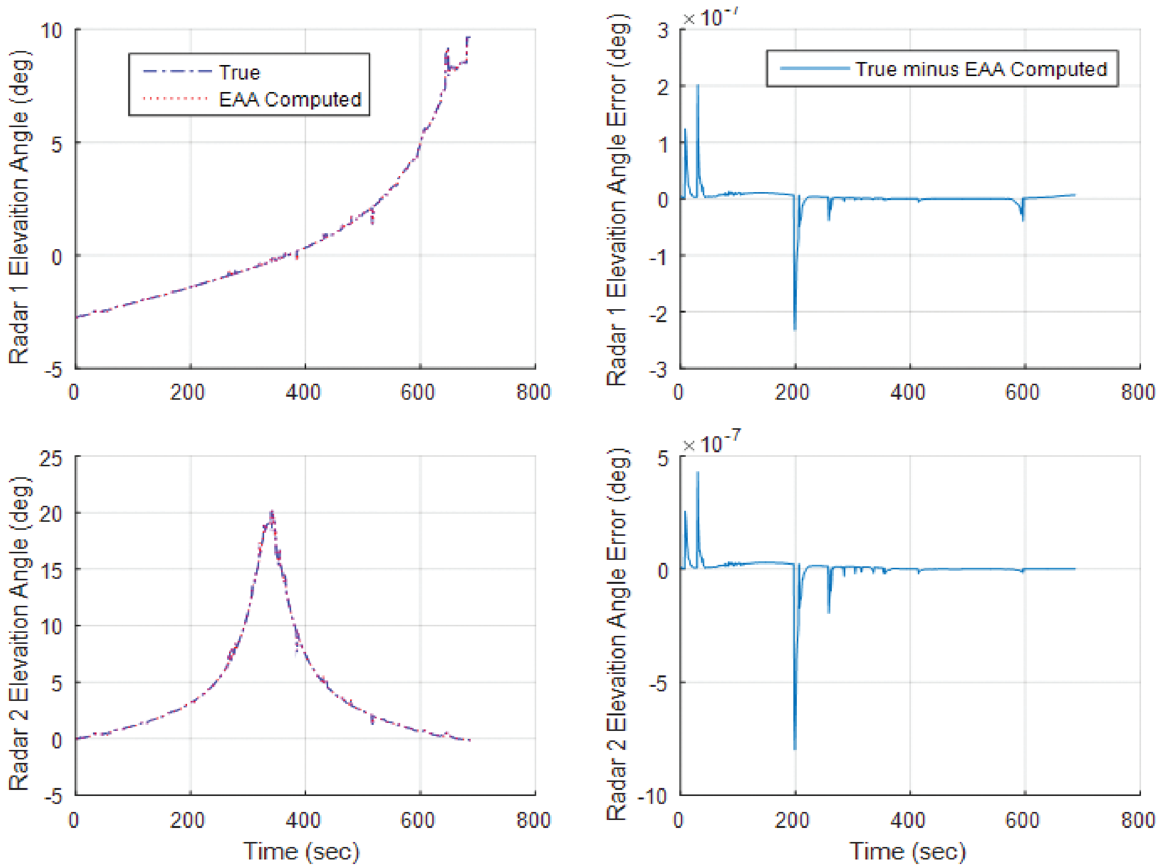
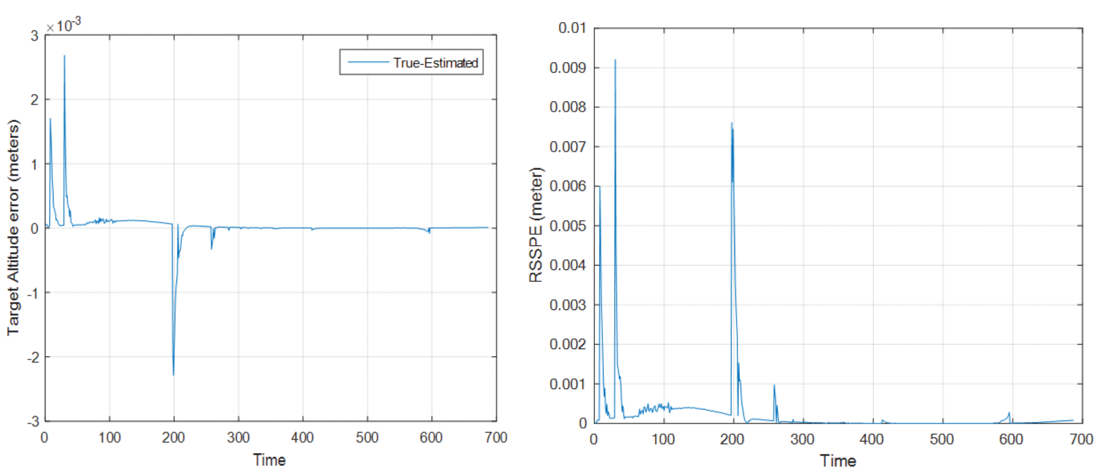


Figure 6. Comparison of true and computed elevation angle - noise free case.



(a) (b)

Figure 7. Target altitude error and RSSPE– noise free case.

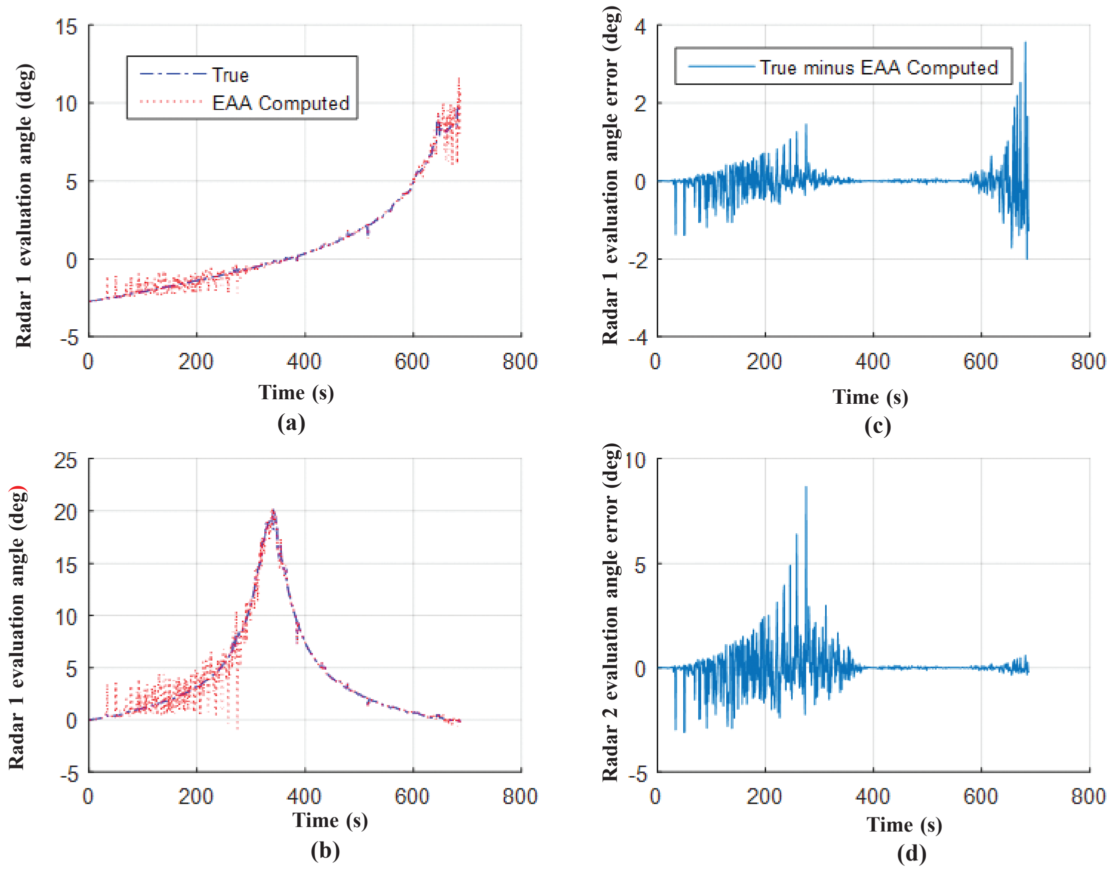


Figure 8. Comparison of true and computed elevation angle – noisy measurements.

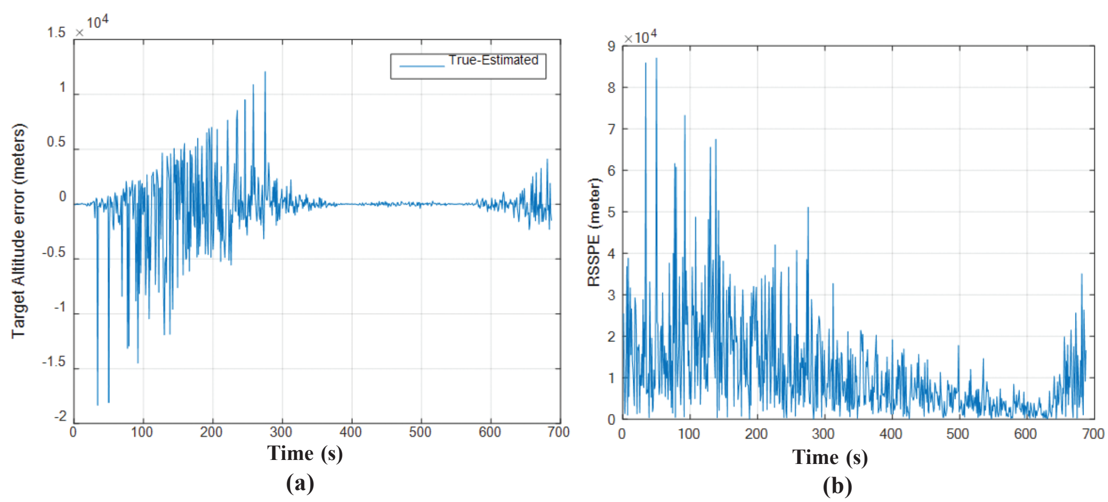


Figure 9. Target altitude error and RSSPE– noisy measurements.

The EAA is futher evaluated with 2D EKF and 3D EKF approaches mentioned in sub-section 2.4.1 and 2.4.2 respectively. Figure 10 shows the comparision of computed angles using these approaches with true value.

It can be seen from Fig. 10 that elevation angles computed using EAA for 3D EKF approach is much accurate than that of computed using 2D EKF approach. The overall elevation angle error are less than 1 degree which shows significant improvment in perfomance as compared to elevation angle obtained directly from noisy measurements. Figure 11(a) shows the altitude error for these approaches.It can be seen from Fig.11(a) that altitude error with 3D EKF is much less as compared to 2D EKF. Figure 11(b) shows the lesser RSSPE error with 3D EKF as compared to 2D EKF. It is recommended to use 3D EKF based EAA method if the radar measurements are synchronous otherwise 2D EKF based EAA is best practicable solution for asynchronous radar measurements. The accuracy of 2D EKF can be further improved by fine tuning of its process noise or measurement noise covariance matrices.

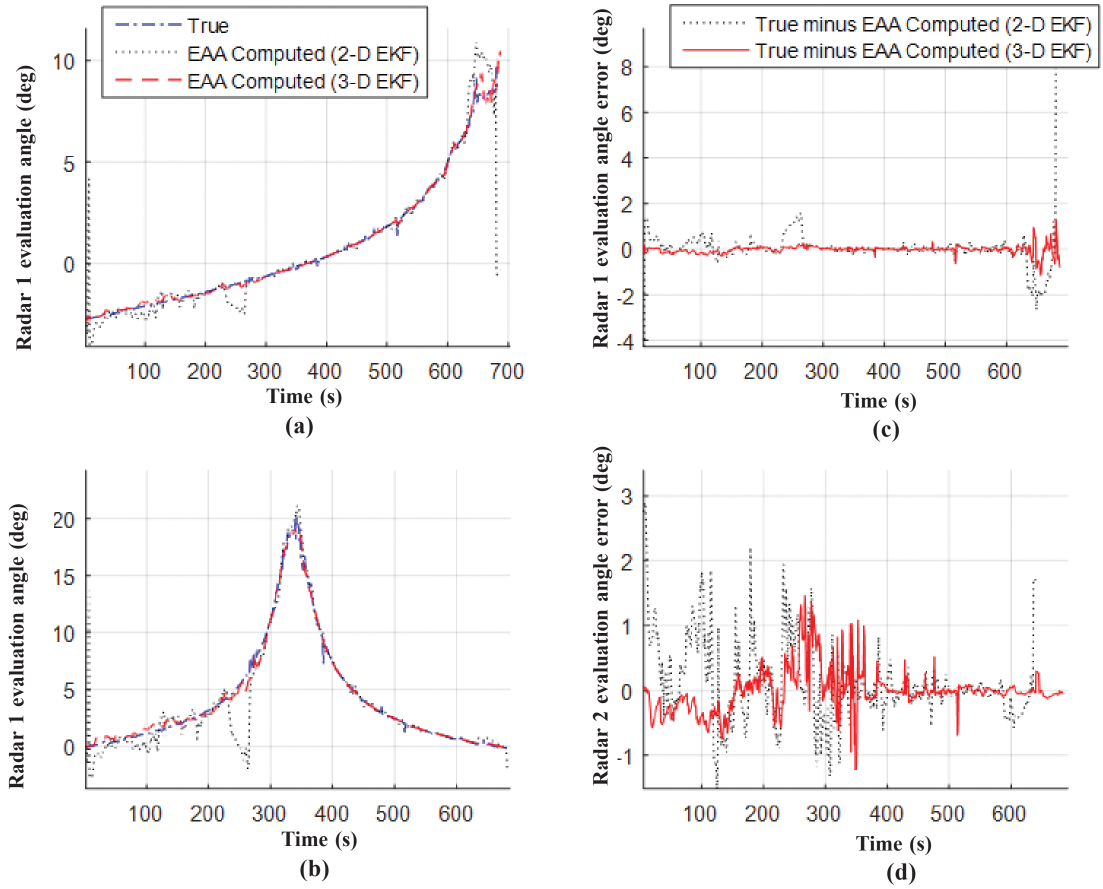


Figure 10. Comparison of true and computed elevation angle (2D/3D EKF approaches).

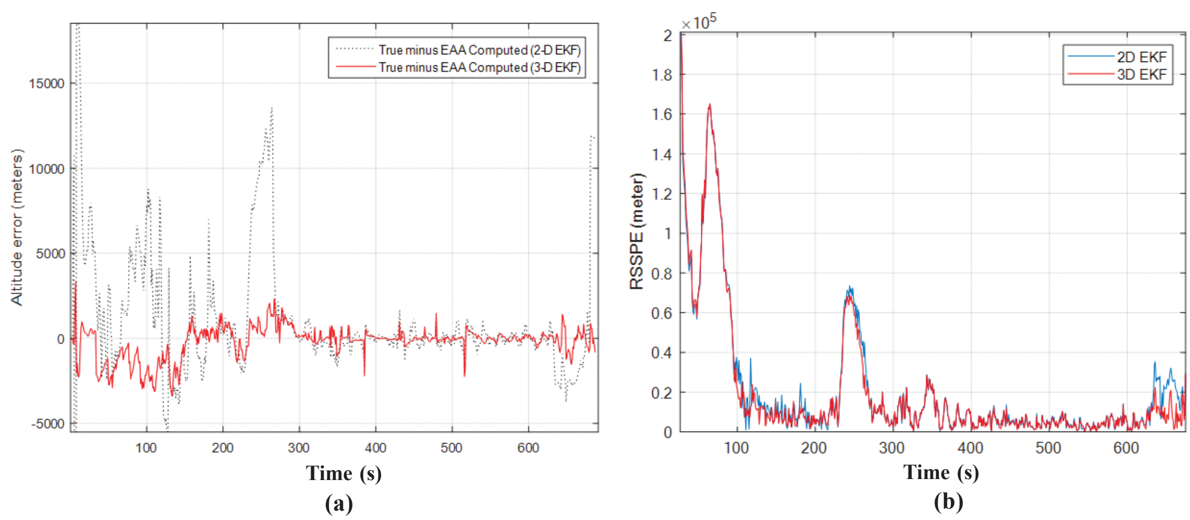


Figure 11. Target altitude error and RSSPE(2D/3D EKF approaches).

3.1 Study on Compuation Speed

The compuation speed of EAA was investigated on MATLAB using tic and toc command for entire simulated data. Figure 12 shows the processing time taken by EAA for every measurement update of radar. It can be seen that processing time is between 5 msec -15 msecwhere as radar measurements update interval is 4000 msec (around 15 RPM which falls in a cateogory of very slow radar). Therefore if multiple targets are process sequentially then around 4000/15 that is 250 targets can be handled by EAA method in real-time. If the EAA code is implemented on C/C++ then process time may be much faster and more than 250 targets can be handled sequentially.

3.2 Comparision with Other Methods

Proposed EAA method is compared with a existing method such as RoL[2]. Figure 13 shows the comparison of target height error for these methods for simulated noise free data. The accuracy of EAA is comparable with existing RoL method. Active localisation method needs slant range

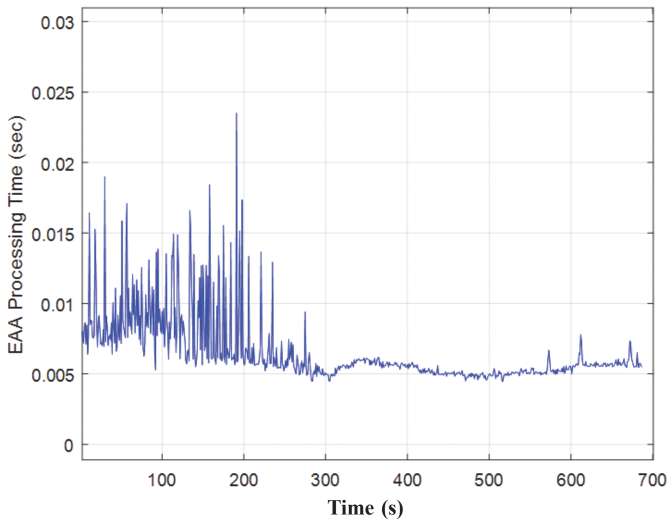


Figure 12. EAA processing time history.

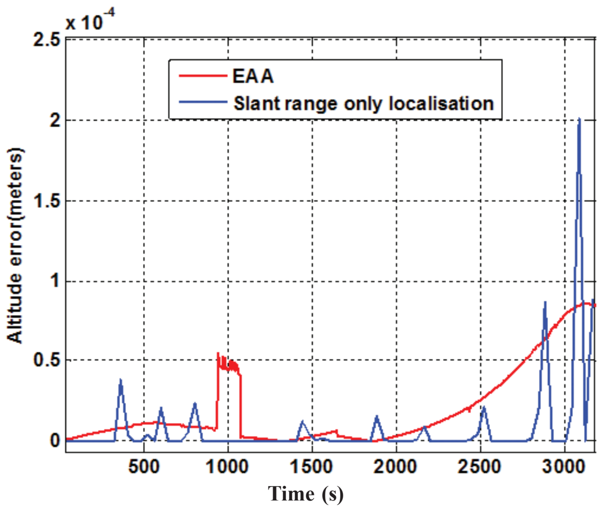


Figure 13. Comparison of EAA and existing RoL method.

measurement from minimum of three contributing 2D radars where as EAA method needs slant range and azimuth measurements from only two contributing radars. Both method uses optimisation technique to estimate the target position. From computational time point of view, it is found that EAA is faster than slant range only localisation method, therefore, more number of targets can be handled by EAA in case of multi-target localisation problem.

3.3 Study with Real Data

The proposed method EAA is also evaluated using various sets of real data from network of ground 2D radars. The estimated height of target is found to be comparable with reference height obtained from on-board GPS data. The results of comparsion are not presented in this paper to maintain the confidentiality of real data. The major challenges encountered while handling real data are summarised below:

- Constant radar biases/scale factor need to be known a priori and shall be removed from radar measurements before applying localisation method.
- Noisy and asycnhronous measurements[29] from radars need to be handled using non-linear filtering technqiue such extended Kalman filter (EKF) before applying localisation method. Interacting multiple model (IMM)[30] shall be used in case of tracking manuevring target.
- To filter the noise, measurement noise covariance (R) shall be obtained from vendor or shall be charactersied using statistical approach/calibaration through various flight sorties. It is found that EKF/IMM without proper tuning can significantly affect the performance of localisation method.
- It is found from monte-carlo study that target-radars relative geometry can affect the accuracy/precision of localisation method. It is therefore recommended to use different localisation methods with some sort of dynamic strategy to select the suitable method based on number of contributing radars, radar accuarcy and target-radars relative geometry.

4. CONCLUSIONS

A new novel method named as EAA is proposed here to compute the target elevation angle using two 2D ground radars and estimate the target position in WGS 84 frame. The advantage of the proposed technique is that it requires only two radars to get accurate position of target. This method can be useful to utilise network of vintage long range 2D radars to determine target position and to fill the gap wherever/whenever target is out of detection range of 3D radars. The technique uses simple logic, easy to implement and suitable for real-time application. From the computational time point of view, it is found that EAA takes very less processing time, therefore, more number of targets can be handled by EAA for multi-target localisation problem. In addition to slant range, this technique requires azimuth measurements which makes it more susceptible to measurement noise if not properly handled using EKF. The method is more accurate for radars which are opposite sides of target and less accurate if they are one side of target. It is therefore recommended to use different localisation methods (EAA, RoL, Trilateration) with some sort of dynamic strategy to select the suitable method based on number of contributing radars, radar accuarcy and target-radars relative geometry.

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His contribution in current study is on design and development of EAA technique¸ its integration with EKF and evaluation using simulated and real data.